Here's a few more problems – these may be a little more challenging but they will help prepare for the exam.

1. If $P(B)=0.60$ and $P(A\cap B)=0.10$, **which is more likely: A|B or B|A?**

Solution:

$$P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{.10}{.60}\approx.1667 \quad \text{and} \quad P(B|A)=\frac{P(B\cap A)}{P(A)}=\frac{.10}{P(A)}$$

but P(A) is unknown. What do we know about P(A)? We can re-write the probability of A as the probability of A and B plus the probability of A and NOT B:

$$P(A)=P(A\cap B)+P(A\cap B^C)=.10+P(A\cap B^C)$$

Because $P(B)=.60$, $P(B^C)=.40$, so we know that $P(A)\le.10+.40$

Thus: $P(A)\le.50$ so $\frac{1}{P(A)}\ge 2$, and $\frac{.10}{P(A)}=(.10)\frac{1}{P(A)}\ge.10\cdot 2=.2$

Basically, we know that because P(A) is no bigger than .5, P(B|A) must be larger than .2
Therefore, we can say with certainty that P(B|A) > P(A|B)

2. If it rains there is a 70% chance that it is windy also. The forecast gives a 25% chance of rain. Therefore, the probability of wind is $.70\cdot.25=.175$ (ie 17.5%) probability of wind.
What is wrong with this reasoning?

Solution:
The problem with the reasoning is that windy with no rain is being completely ignored. We can say the prbability of wind AND rain is 17.5%, but $P(Wind)=P(Wind\cap Rain)+P(Wind\cap No\,Rain)$.

3. At a certain factory, units are produced on the assembly line one after the other. Sometimes there is a glitch in the system and the factory produces defective items.
A defective item is produced after a working item with probability 0.1%. A defective item is followed by another defective item with probability 95%. Assume that the machines are checked every night so that the first item of the day is a working item with 100% probability.
- **What is the probability that the second item of the day is working?**

We know the first item is working from the problem, so we can base this off the probability that an item works given the previous item was working
$P(Item\,2\,is\,Working)=P(Working\,Item|Previous\,Item\,was\,Working)=99.9$

- **What is the probability that the second through 5th items are all working?**

We want to know the probability of 4 working items in a row given the first item was working.
The probability the second works is .999

The probability the third works if the second works is .999
So on for #4 and #5, so the probability that #2-#5 all work is $.999^4 \approx .996$

- **What is the probability that the second item is defective but the 10 after it are all working?**

We have 1 defective item following a working item
1 working item following a defective item
and then 9 more working items each following a working item
These events happen with probabilities 0.001, .05, and .999 respectively
Then the probability that all of this happens is $.001 \cdot .05 \cdot .999^9 \approx .00004955$

- **What is the probability that the first 4 items of the day will be working, defective, working, and working (in that order)?**

We have 1 working item automatically – there's 100% chance the first item of the day will be working.
Then there is a .001 chance the second item is defective
The probability the third is working given the second was defective is .05
Finally the probability that the fourth is working given that the third was working is .999
So P(working, defective, working, working) = $.001 \cdot .05 \cdot .999 = .00004995$

- **If item #3 is defective, what is the probability that item #2 was working?**

For this problem, a tree diagram is useful:
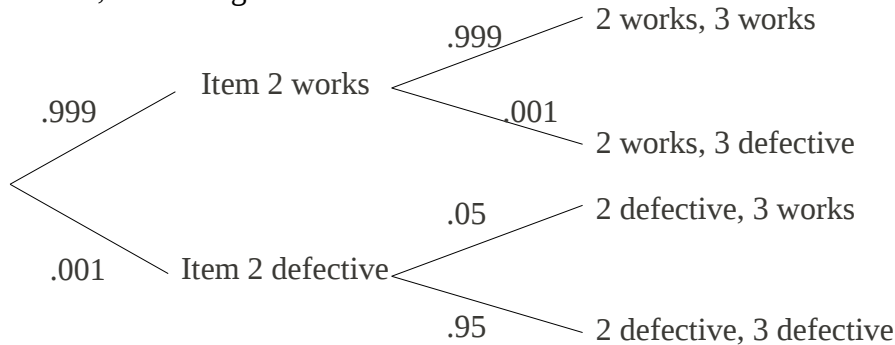
So we can see that the probability that 2 is working GIVEN that 3 is defective is found by:

$$P(2\,works|3\,defective) = \frac{P(2\,works \cap 3\,defective)}{P(3\,defective)} = \frac{.999 \cdot .001}{.999 \cdot .001 + .001 \cdot .95} \approx .5126$$

- **If the company produces 50 items today, what is the probability that the company will only produce 1 defective item today?**

There are 49 disjoint cases: The defective item could be item #2 through #50
The probability of the defective item being #2 is:
Probability of 2 defective after 1 is working (.001)
times Probability that 3 is working after 2 is defective (.95)
times Probability that 4-50 are working after 3 is working ($.999^{47}$)
= .0000477

You can see that the probability of item #3 being defective is the same – you will take the product of the exact same numbers. That is also true up through item #49 being defective

The exception is if item #50 is the defective item. In this case the probability is
$.999^{48} \cdot .001 \approx .000953$

So by adding up all of these probabilities we have:

$P(1\,out\,of\,50\,is\,defective)=48 \times .0000477+.000953 \approx .003243$

- **Is the binomial distribution appropriate to predict the # of defective items produced? Why or why not?**

Absolutely not! Because the probability of producing a defective item depends on the item before it, these are not independent Bernoulli trials, so we cannot use a Binomial model for this problem.